The Sensitivity Analysis of the Method for Identification of Bearing Dynamic Coefficients

Łukasz Breńkacz and Grzegorz Żywica

Abstract This article presents the sensitivity analysis of the method for determination of mass, damping and stiffness coefficients using the impulse excitation technique for a rotor-bearing system. Such an experimental approach is an adequate tool for the estimation of 24 dynamic coefficients, that is 4 damping coefficients, 4 mass coefficients and 4 stiffness coefficients for each bearing. As yet, the literature is exclusive of any researches into the sensitivity of this experimental method itself. However, the influence of several parameters (e.g. supply pressure, bearing geometry, etc.) on the calculation results concerning bearing dynamic coefficients had already been examined in detail. The preparation of the numerical model of the rotor made it possible to assess how influential are the input parameters—such as position and angle of an excitation force or movements of the sensor heads used to measure the displacements of bearing journals—to the results. The potential impact of changing parameters, such as stiffness of rotor material, its unbalance or its geometry, on the values of calculated stiffness, damping and mass coefficients in tested rotor-bearing system was also verified. The paper presents the calculation results of dynamic coefficients for the bearings along with their relative errors. It was shown how the calculated values change according to the different input parameters. The excitation signals and the corresponding system responses were also provided. Moreover, the article contains information on how to enhance the accuracy of calculations.

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1 Introduction

A number of methods for determining the stiffness and damping coefficients of bearings on the basis of experimental tests have been described in scientific literature. The impulse response method for identification of mass, damping and stiffness coefficients of rotor-bearing system was described in the article [1]. It presents the calculation algorithm and its results for different identification ranges and the corresponding measurement error. The Impulse Excitation Technique (IET)—based on calculations in the frequency domain, is considered to be the most economically efficient and giving one of the most reliable results [2]. It is a linear method.

There are a number of experimental methods for identifying bearing dynamic coefficients [3, 4]. The stiffness and damping coefficients vary with the rotational speed [5]. When it comes to slide bearings, they are inherently non-linear in nature, which means that their dynamic behavior may change at constant rotational speed, as well as they depend on the excitation force [6]. Dynamic coefficients also change with the ambient temperature, bearing supply pressure and load [7]. The computations of bearing force coefficients are often used for the bearings with unknown design parameters and the ones of complex structure [8].

Tiwari and Chakravarthy [9] proposed to improve the algorithm for the estimation of stiffness and damping coefficients of bearings in such a way that it additionally enables estimation of the residual unbalance of a rotor. Nowadays, various works are under way aimed at the modification of experimental methods in order to achieve an increased accuracy. Miller and Howard [10] described the method for the identification of bearing rotor-dynamic coefficients using the extended Kalman filter. The calculations were carried out using the linearized stiffness and damping coefficients in rotor-bearing systems, considering the noise and unbalance. The method was successfully used to assess the main stiffness coefficients, whereas the cross-coupling damping coefficients were calculated with the lower accuracy. Application of the Monte Carlo method for investigation of dynamical parameters of rotor supported by magnetorheological squeeze film damping devices was shown in [11]. The new way of identification of the stiffness and damping characteristics of bearings using phase-plane diagrams was presented in the publication [12]. The authors highlight that a reliable assessment of bearing dynamic coefficients is a major challenge, particularly in non-linear systems. They claim that there is no single universal model for calculation of system characteristics because their identification depends on measurement data and reference model.

Numerical determination of dynamic coefficients in the case of foil bearings can be a very difficult task because of their complex structure [13]. The results of the experimental identification of these coefficients for the large-diameter hybrid foil bearing were presented in paper [14]. This article contains dynamic characteristics of hybrid (hydrodynamic + hydrostatic) foil bearing with a diameter of 101.6 mm and a length of 82.6 mm. The stiffness coefficients were determined using two different methods: first, with a quasi-static method based on the load-deflection curves in the time domain; second, with the frequency domain impulse response method. The damping coefficients were obtained using only the impulse response method. The stiffness coefficients values from both methods were close to each other. The article [15] presents the experimental studies on hybrid gas bearing, characterized by a complex design of foil, which is quite tolerant to high supply pressure. The bearing utilized two lubricating films: hydrostatic and hydrodynamic. The paper [16] presents the calculation results of aerostatic radial bearings on the example of Bently Nevada test rig. The tests with various types of bearings were carried out in order to analyze their static and dynamic characteristics. Only the main stiffness and damping coefficients were calculated. The influence of cross-coupling stiffness and damping parameters on the results was assessed on the basis of numerical simulations. During the calculations, the mass matrix elements were considered as known. The stiffness and damping matrixes had to be calculated. The authors recommend that the main coefficients should be estimated at the beginning of the experiment, and then the cross-coupling coefficients, in the next step, since they are more vulnerable to errors. Another example of experimental identification of aerostatic bearing was shown in [17]. The numerical models for calculating stiffness and damping parameters of bearings are not only used for radial bearings, they also continue to make a successful contribution to axial bearing investigations. Experimental identification of stiffness and damping coefficients of an axial foil bearing was shown in the article [18].

2 The Calculations of Bearing Dynamic Coefficients

The numerical model of the rotor, which have been used for the sensitivity analysis of the method, was created on the basis of the test rig called SpectraQuest Machinery Fault and Rotor Dynamics Simulator. This test rig is located at the Szewalski Institute of Fluid-Flow Machinery PAS, in Gdańsk. The numerical model was created in the Samcef Rotors software (Fig. 1). The model consists of a shaft rotating at 2800 rpm and bearings modeled using the stiffness and damping coefficients in the orthogonal and cross-coupling directions. The rotor during the

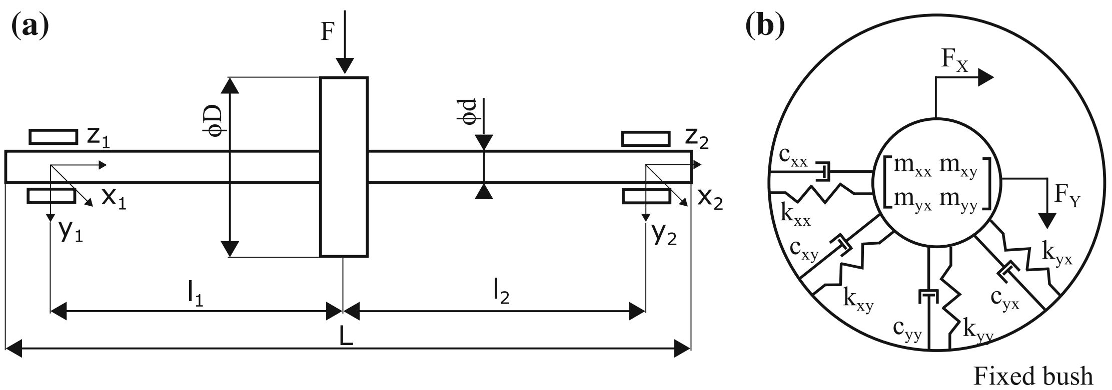

Fig. 1 **a** Rotor model prepared in the Samcef Rotors software, **b** Bearing model

simulation had been excited by a known value of excitation force, in the central part, in the X direction. Then the vibration amplitude of the rotor was measured in each bearing in X and Y directions. Then, the simulation was repeated, this time using the same excitation force, but model has been excited in the Y direction. In this case, the vibration amplitudes of the rotor in the bearings in the direction of X and Y were measured. When the vibration in the X direction in the first bearing after excitation in the Y direction was measured, signal has been described as $D^1{}_{XY}$. Based on the excitation signals and the response signal generated by the Samcef Rotors program the stiffness, damping and mass coefficients of the bearings were calculated. Because the stiffness and damping coefficients were defined in the Samcef Rotors program, calculated coefficients can be compared with their actual values. Taking into consideration different parameters, it was possible to compare results with expected values and calculate the relative error.

The calculations are carried out for two bearings simultaneously. Indexes ‘1’ and ‘2’ stand for the first and the second bearing. The index ‘i’ designates frequency range. The first of double index indicates the direction of the system response, the second index indicates the direction of the excitation force. For example, $D^1{}_{YX}$ element defines the vibrations of the rotor at the first bearing in the X direction after excitation in the Y direction. The least squares method is used to solve the equations and it requires the signal to be represented in the frequency domain. This is achieved by Fourier transform of the force signal and the system response from the time domain. In order to obtain the dynamic compliance the signal representing the displacement in the bearing is multiplied by the signal representing the excitation force. Flexibility is defined as the inverse of mechanical impedance. The least squares method can be applied to the Eq. (1). In this equation, A_i is a matrix in which the signals are elements of flexibility matrix and frequency vector. It was formed by decomposition of real and imaginary part (2). Matrix I is defined by Eq. (3), while the matrix Z consists of stiffness, damping and mass coefficients of the rotor-bearing system (4). If we assume that the parameters are obtained for 100 frequency samples (it is determined by the index (i) the matrix A will have dimension 800 × 12, matrix I will have dimension 800 × 2, while the matrix Z will have dimension 2 × 12.

$$Z = (A_i^T A_i)^{-1} A_i^T I_i, \tag{1}$$

$$A_i = \begin{bmatrix} \mathrm{Re}\left[F_i^1 \cdot \begin{bmatrix} 1 & 0 & 0 & 0 & -(\omega_i)^2 & 0 & 0 & 0 & \omega_i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -(\omega_i)^2 & 0 & 0 & 0 & \omega_i & 0 & 0 \end{bmatrix}\right] \\ \mathrm{Re}\left[F_i^2 \cdot \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & -(\omega_i)^2 & 0 & 0 & 0 & \omega_i & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -(\omega_i)^2 & 0 & 0 & 0 & \omega_i \end{bmatrix}\right] \\ \mathrm{Im}\left[F_i^1 \cdot \begin{bmatrix} 1 & 0 & 0 & 0 & -(\omega_i)^2 & 0 & 0 & 0 & \omega_i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -(\omega_i)^2 & 0 & 0 & 0 & \omega_i & 0 & 0 \end{bmatrix}\right] \\ \mathrm{Im}\left[F_i^2 \cdot \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & -(\omega_i)^2 & 0 & 0 & 0 & \omega_i & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -(\omega_i)^2 & 0 & 0 & 0 & \omega_i \end{bmatrix}\right] \end{bmatrix}, \tag{2}$$

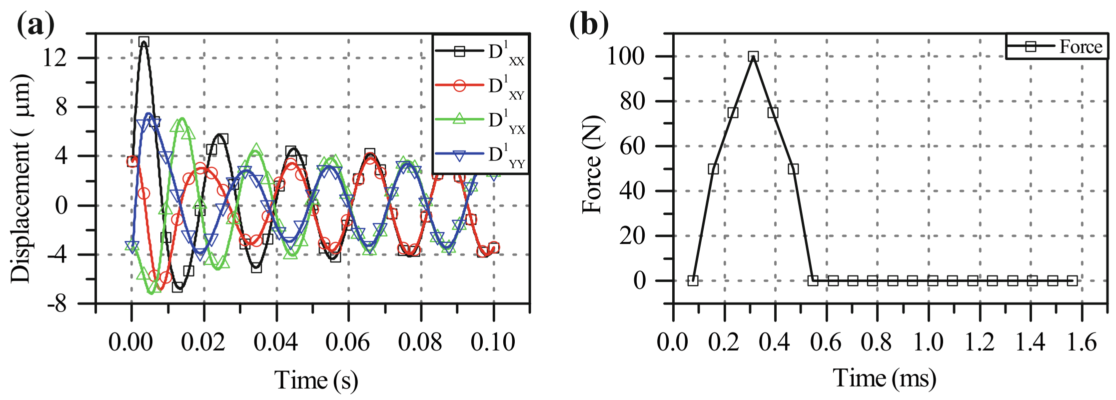

Fig. 2 **a** Graph of the bearing bush displacement versus time after excitation of the rotor in its central part, **b** Force in the X direction versus time

$$I_i \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (3)$$

Figure 2 a contains the graph of bearing displacement versus time, drawn up after the rotor was excited in its central part. It is composed of the component resulting from the excitation and the constant component linked to the existence of rotor unbalance. In the next step of the algorithm, the constant component mentioned in the previous sentence should then be deducted from the obtained signal. Figure 2b presents the signal which represents the excitation force. It was defined in the Samcef Rotors software as a vector containing force values at successive time steps. The numerical simulation was carried out for the time steps having the same resolution of 1/12800 [s].

3 Sensitivity Analysis of the Numerical Model

The carried out sensitivity analysis was not aimed at looking into all the possible parameters but to indicate the parameters which substantially affect the results of the calculation. Figure 3 contains the flowchart according to which the sensitivity assessment of the experimental identification method used to be performed. On its left side one can see the parameters which were altered during the determination of bearing dynamic coefficients. The calculation algorithm, used in the method described here, can be seen in the central part of the flowchart. The right-hand side of Fig. 3 shows that a summary of the bearing coefficients results for various input parameters together with the calculation error estimation have been created, at the last stage of the sensitivity analysis. As the stiffness and damping coefficients are known for the rotor concerned, the calculated values and real values of the bearing coefficients can be directly compared (Table 1).

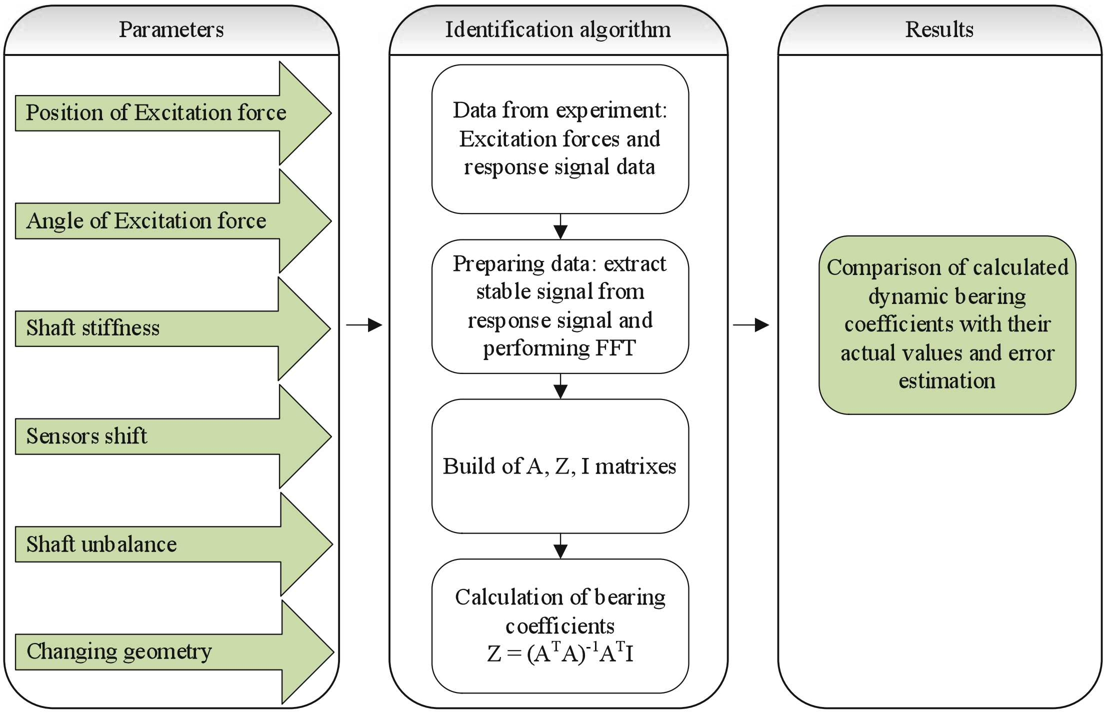

Fig. 3 Sensitivity analysis flowchart

Table 1 Parameters of the numerical model

Parameter	Value
Length	L = 0.92 m
Distance between bearings	$2 \times l_1 = 0.58$ m
Rotor shaft diameter	d = 19.05 mm
Disk diameter	D = 152.4 mm
Young's modulus	$E = 205 \times 10^9$ Pa
Poisson's ratio	0.3
Material density	$\rho = 7800$ kg/m^3

4 Calculations Concerning the Reference Model

The signals generated using Samcef Rotor program were used to calculate the stiffness, damping and mass coefficients of the rotor-bearing system. The model of symmetric rotor was considered, but the stiffness and damping coefficients (including cross-coupling ones) have different values. In the hydrodynamic bearings, the cross-coupling part of damping coefficients C_{xy} and C_{yx} has got the same values [4], in this case they have different values in order to show how the method works. Since the values of stiffness and damping coefficients were taken from the numerical model of an existing rotor, it was possible to compare them directly and, on that basis, the calculation error of force coefficients was estimated.

The calculation results together with the corresponding relative errors are shown in Table 2. It has to be mentioned that the operations were carried out upon the

Table 2 List of actual and calculated stiffness, damping and mass coefficients for both bearings, reference case. The relative errors were also listed

	Stiffness coefficients N/m							
	k^1_{xx}	k^1_{yy}	k^1_{xy}	k^1_{yx}	k^2_{xx}	k^2_{yy}	k^2_{xy}	k^2_{yx}
Values	500000	450000	250000	240000	550000	470000	270000	260000
Calculated	498232	450488	248338	240544	548107	470504	268288	260583
Error %	0.35	0.11	0.66	0.23	0.34	0.11	0.63	0.22
	Damping coefficients N · s/m							
	c^1_{xx}	c^1_{yy}	c^1_{xy}	c^1_{yx}	c^2_{xx}	c^2_{yy}	c^2_{xy}	c^2_{yx}
Values	500	550	250	260	550	560	260	270
Calculated	507.5	547.7	259.4	258.8	558.1	558.3	269.2	270.7
Error %	1.50	0.42	3.76	0.46	1.47	0.30	3.54	0.26
	Mass coefficients kg							
	m^1_{xx}	m^1_{yy}	m^1_{xy}	m^1_{yx}	m^2_{xx}	m^2_{yy}	m^2_{xy}	m^2_{yx}
Actual	2.423	2.423	0.000	0.000	2.423	2.423	0.000	0.000
Calculated	2.338	2.404	–0.071	–0.029	2.497	2.451	–0.035	–0.053
Error %	3.49	0.77	–	–	3.07	1.18	–	–

signal only after eliminating the constant component. The rotor rotational speed was 2800 rpm. The signal related to the rotational speed, which was represented in the frequency domain falls into the resonance range of the rotor. The relative error estimated for the bearing stiffness coefficients does not exceed 0.7 %. This is a quite accurate estimation, especially given the fact that the error of calculating the stiffness coefficients in the main coordinates (which are more significant in further modeling of the dynamics of the system) does not exceed 0.4 %. The error resulting from the calculation of damping coefficients is small and does not exceed 3.8 %. This error appears only in the calculation of the cross-coupling part of damping coefficients. The error concerning the damping coefficients in main coordinates is up to about 1.5 %. The weight of the shaft modeled in Samcef Rotors program is 4.845 kg. The calculated weight of the shaft based on described algorithm and numerical model of the shaft is 4.845 kg. This mass was calculated by adding the mass coefficients on the main diagonal (*XX* and *YY*), and dividing this value by two. Interpretation of mass coefficient values is as follows: the mass coefficient $m^1{}_{xx}$ is the mass of part of the shaft involved with the vibration in the *X* direction allocated to the first bearing after excitation in the *X* direction. In contrast, the mass coefficient in $m^1{}_{yy}$ is the mass of part of the shaft involved with the vibration in the *Y* direction allocated to first bearing after excitation in the *Y* direction. Dividing the sum of the weight coefficients by two to calculate the mass of the shaft is applied because the system was excited twice. Cross-coupling mass coefficients (*XY YX*) should be close to zero. The error of calculating weight coefficient is 0.003 %. These results mean that the rotor weight 4.845 kg can be estimated with an accuracy of ±0.0001 kg.

The subsequent sensitivity analysis consisted in changing the stiffness of rotor material. The value of Young's modulus has been significantly increased, namely by 100 times. This test was intended to check whether material stiffness affects the calculation results. After this test, it turned out that the equivalent results were obtained in two cases. This means that the value of modulus of elasticity for steel, corresponding to the most frequently encountered shaft materials, does not affect calculation results of stiffness, damping and mass coefficients of rotor-bearing system.

5 Impact of Unequal Excitation Distribution on Two Bearings

The algorithm described here assumes a symmetrical distribution of excitation force between two bearings. This assumption requires rotor excitation to be performed in its central part, however it is not possible to implement for each type. In the course of model testing it turned out that the small shift of excitation force position (as small as by five per cent of the distance between bearings) may lead to identification of bearing dynamic coefficients with an error rate around a dozen percent. Appropriate unsymmetrical distribution of the excitation force between two bearings results in a satisfactory outcome of such calculations. Note that in the example described above, the shift of force from the center of the shaft was 30 mm (denoted as "s"). This represents 5 % of the shaft length between bearings. The results were compared with those obtained earlier for the force F_{ref} aligned on the center of the rotor's shaft.

In the algorithm applied for computing the bearing dynamic coefficients one can introduce modifications in order to enhance calculation accuracy. Force values related to the bearings should be calculated on the basis of geometrical proportions. Adjustment of the excitation force shall be to multiply every element of the force vector by the expressions (4, 5). The force values divided proportionally and not proportionally between the first and the second bearing were taken into account in the Eq. (2). F_1 denotes the value of force applied to first bearing, while $F_{1\mathrm{shifted}}$ represents the same force after its proper transformation. The l_1 and l_2 dimensions define the distances of the bearing supports from the center of the shaft. The values denoted as l_{1f} and l_{2f} specify the distances of the force to the bearings.

$$F_{1\mathrm{shifted}} = F_1 \cdot \frac{l_{2f}}{2 \cdot l_1}, \tag{4}$$

$$F_{2\mathrm{shifted}} = F_2 \cdot \frac{l_{1f}}{2 \cdot l_2}. \tag{5}$$

In the case when in the algorithm the force was not shifted properly the relative errors were up to 12, 15.5, and 1.73 % for stiffness, damping and mass coefficients, respectively. After shifting the excitation force the relative errors concerning dynamic coefficients has changed radically. They were as follows: below 0.73 % for stiffness, below 3.73 % for damping and around 0.2 % for mass. In this place it should be mentioned that the calculation accuracy for main stiffness and damping coefficients is higher than it is the case with the cross-coupling coefficients.

It is worth noting that a shaft mass is often known parameter before starting the calculation process, contrary to stiffness and damping coefficients. If this is the case, once all bearing dynamic coefficients have been calculated, the calculation error corresponding to mass coefficients can be assessed. This enables to reject incorrect estimates at an early stage of the identification. The expected values of main mass coefficients (in *xx* and *yy* directions) are equal to half of shaft mass in the tested model. After the calculations were carried out with not shifted excitation force, one can say that these expected values of mass coefficients and calculated ones varied considerably (approximately 12.4–18.4 %). Then, considering the uneven division of the excitation force in the calculations, the relative errors were ranging from 2.8 to 6.1 %.

6 Change of Excitation Force Direction and Its Consequences

The experimental studies are carried out by double exciting the rotating shaft using an impact hammer in a direction transverse to the axis of rotation of the rotor. On the basis of the example provided in the previous chapter, it may be said that in order to obtain correct results of dynamic coefficients identification, it is necessary to adopt good definition of excitation force. This chapter describes the case in which a hammer hits the rotating rotor at a certain angle in relation to the intended orientation. In order to achieve this effect, the impact marked $F_{x\alpha}$ was introduced into the numerical model. It designates an impact hammer hit at an angle of 15° to the intended direction. The impact marked F_y means that the impact acted along the Y axis. Figure 4a presents the diagram of an excitation force acting at a certain angle in relation to the intended orientation while Fig. 4b shows the system response for the bearing number 1. Only the system responses in X and Y directions after the excitation in X direction were presented. They are characterized by the highest difference in the results. The notation $D^1{}_{XX}$ ref and $D^1{}_{YX}$ ref indicates system response for the force F_x acting in X direction. The signals $D^1{}_{XX}$ and $D^1{}_{YX}$ are generated after excitation of the system by means of the force $F_{x\alpha}$ applied in an angle 15°. It should also be emphasized that the angle of application of the excitation force is quite high and should be minimized, to the greatest possible extent, when conducting experimental tests.

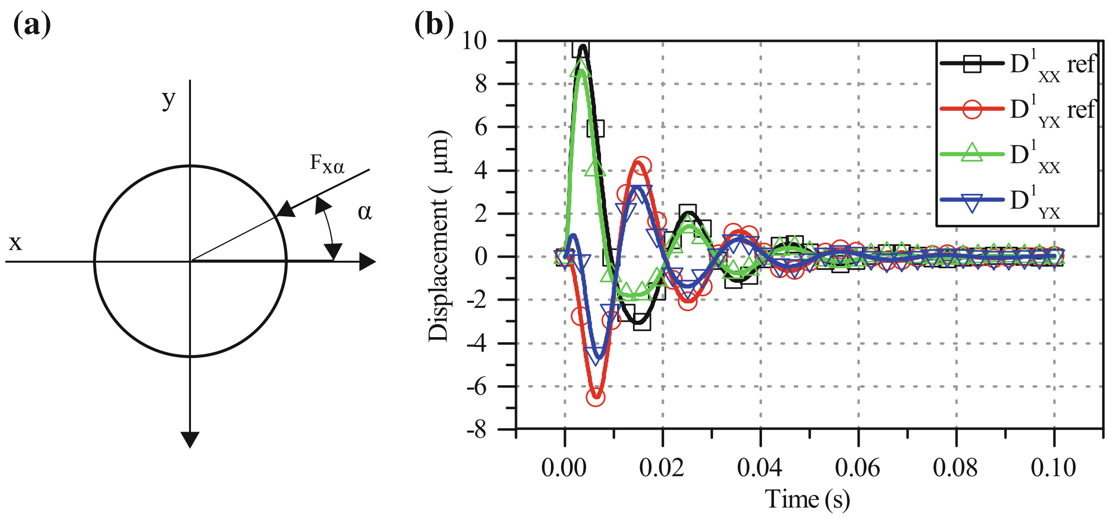

Fig. 4 **a** Diagram presenting the force applied at a certain angle **b** System response signals in *X* and *Y* directions after its excitation by using force at an angle of 0° and 15°

Table 3 List of actual and calculated stiffness, damping and mass coefficients for both bearings, case with the excitation force applied at an angle of 15°

	Stiffness coefficients N/m							
	k^1_{xx}	k^1_{yy}	k^1_{xy}	k^1_{yx}	k^2_{xx}	k^2_{yy}	k^2_{xy}	k^2_{yx}
Values	500000	450000	250000	240000	550000	470000	270000	260000
Calculated	516049	383855	257221	106858	567699	398520	277880	113519
Error %	3.21	14.70	2.89	55.48	3.22	15.21	2.92	56.34
	Damping coefficients N · s/m							
	c^1_{xx}	c^1_{yy}	c^1_{xy}	c^1_{yx}	c^2_{xx}	c^2_{yy}	c^2_{xy}	c^2_{yx}
Values	500	550	250	260	550	560	260	270
Calculated	523.9	478.5	267.8	123.4	576.2	486.5	277.9	121.7
Error %	4.78	13.00	7.12	52.54	4.76	13.13	6.88	54.93
	Mass coefficients kg							
	m^1_{xx}	m^1_{yy}	m^1_{xy}	m^1_{yx}	m^2_{xx}	m^2_{yy}	m^2_{xy}	m^2_{yx}
Values	2.423	2.423	0.000	0.000	2.423	2.423	0.000	0.000
Calculated	2.426	2.424	–0.071	–0.065	2.591	2.462	–0.034	–0.072
Error %	0.16	0.07	–	–	6.95	1.62	–	–

Table 3 lists the results of stiffness, damping and mass coefficients calculated on the basis of system response signals generated after excitation of the system in X direction by the force applied at an angle of 15° (which is identified by "α" in Fig. 4 a. In the calculation algorithm the whole of the value of the force is treated as if the excitation was introduced at an angle of 0°. It turns out that the relative errors corresponding to stiffness coefficients, damping coefficients and shaft mass were around 56, 55 and 2.2 %, respectively. It should be noted in this respect that the relative errors for some coefficients do not exceed 3 %, so not all of the results are burdened with a considerable error. The highest levels of relative errors resulted from the identification of cross-coupling coefficients *YX*. The calculation of *YY* coefficients also produced significant errors. The relatively small errors were observed in the calculation of *XX* and *YY* coefficients.

7 The Impact of Sensors Shifting

The identification of bearing dynamic coefficients is carried out on the basis of excitation force signal and system response coming from actual location of bearings. Since sensor placement inside bearing support is usually impossible, one has to check how the placement of sensors right next to the bearing supports affects the results of the calculations. The ideological diagram of arrangement of bearing supports and measurement sensors was presented in Fig. 5. The s_1 and s_2 dimensions define the distances of the bearing supports from the center of the shaft. The values denoted as l_{1p} and l_{2p} specify the distances of the sensors to the center-point of the system. The rotor length was marked with the letter L.

The calculations that were carried out after the measuring points had been moved toward the center of the shaft by 30 mm (it corresponds to 5 % of the distance between the bearing supports) show that the relative errors relating to stiffness, damping, and mass coefficients amounted to about 2 %. In order to enhance the accuracy of calculations, Eqs. 6 and 7 shall be applied. When system response signal measured at the placement of the sensors was multiplied by the expressions listed below, the calculation results concerning bearing dynamic coefficients were consistent with those generated on the basis of the signals measured in the center of bearing supports.

$$\begin{bmatrix} XX_1 & YY_1 \\ XX_2 & YY_2 \end{bmatrix} = \frac{1}{s_1 + s_2} \cdot \begin{bmatrix} s_2 + l_1 & s_1 - l_1 \\ s_2 - l_2 & s_1 + l_2 \end{bmatrix} \begin{bmatrix} XX_{1p} & YY_{1p} \\ XX_{2p} & YY_{2p} \end{bmatrix}, \tag{6}$$

$$\begin{bmatrix} XY_1 & YX_1 \\ XY_2 & YX_2 \end{bmatrix} = \frac{1}{s_1 + s_2} \cdot \begin{bmatrix} s_2 + l_1 & s_1 - l_1 \\ s_2 - l_2 & s_1 + l_2 \end{bmatrix} \begin{bmatrix} XY_{1p} & YX_{1p} \\ XY_{2p} & YX_{2p} \end{bmatrix}. \tag{7}$$

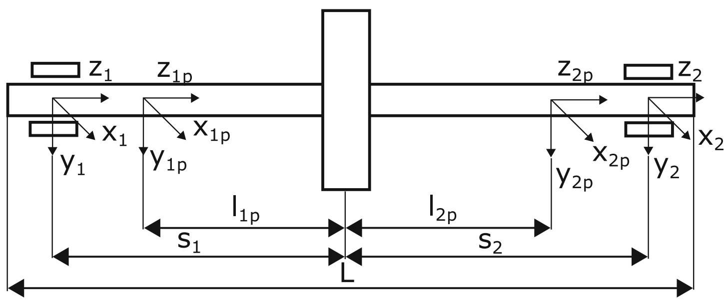

Fig. 5 Ideological diagram on which are marked points of measurement relating to the bearings and the distances taken into account during calculation of bearing dynamic coefficients

8 Case of Asymmetrical Rotor

It often happens that it is necessary to identify bearing dynamic coefficients for a system in which the bearings are at different distances from the ends of the shaft. The objective of this chapter is to put into analysis the calculation results obtained for an exemplary asymmetrical rotor. For this purpose, one shaft end has been shortened by 60 mm. Schematic view of the rotor is presented in Fig. 6. The rotor dimensions were as follows: $l_1 = l_2 = 290$ mm, $L_1 = 460$ mm, and $L_2 = 400$ mm. The system response at the bearing no. 1 after applying the excitation force between the bearings was shown in Fig. 7. The change in the rotor's geometry described above caused that the rotor mass has decreased slightly to 4.71 kg.

Table 4 lists the stiffness, damping and mass coefficients in the case of asymmetrical rotor. The relative errors concerning the stiffness coefficients and damping coefficients do not exceed 0.6 and 4 %, respectively. The shaft mass which was 4.71 kg was determined as 4.709 with accuracy of 0.012 %. Also in this case, all the main coefficients were identified with higher accuracy than the cross-coupling ones.

Fig. 6 Schematic view of the asymmetrical rotor

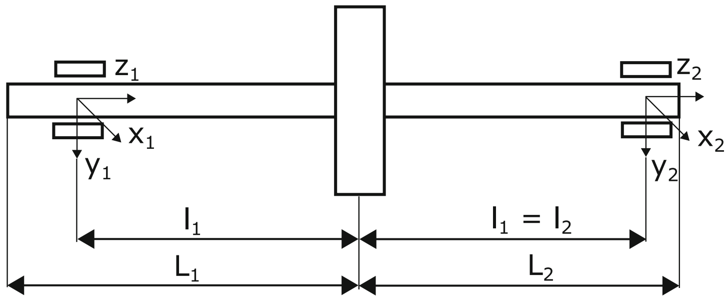

Fig. 7 Response signal of the asymmetrical system

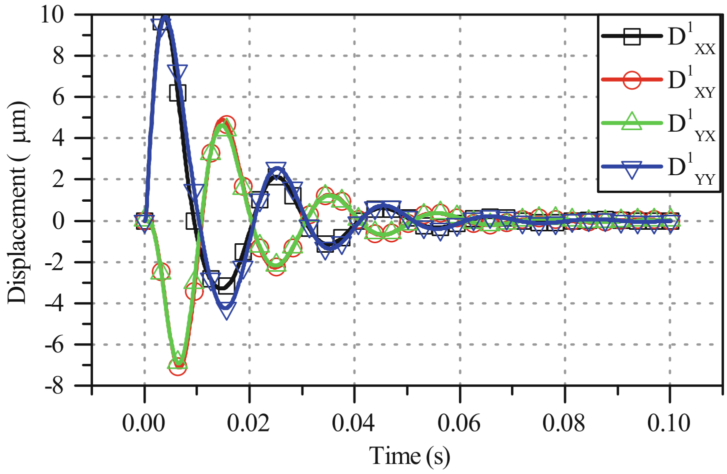

Table 4 List of actual and calculated stiffness, damping, and mass coefficients for both bearings, asymmetrical rotor case

	Stiffness coefficients N/m							
	k^1_{xx}	k^1_{yy}	k^1_{xy}	k^1_{yx}	k^2_{xx}	k^2_{yy}	k^2_{xy}	k^2_{yx}
Values	500000	450000	250000	240000	550000	470000	270000	260000
Calculated	498198	450495	248331	240585	548158	470442	268370	260557
Error %	0.36	0.11	0.67	0.24	0.33	0.09	0.60	0.21
	Damping coefficients N · s/m							
	c^1_{xx}	c^1_{yy}	c^1_{xy}	c^1_{yx}	c^2_{xx}	c^2_{yy}	c^2_{xy}	c^2_{yx}
Values	500	550	250	260	550	560	260	270
Calculated	506.9	547.2	259.7	259.5	557	558.6	268.7	271.9
Error %	1.38	0.51	3.88	0.19	1.27	0.25	3.35	0.70
	Mass coefficients							
	m^1_{xx}	m^1_{yy}	m^1_{xy}	m^1_{yx}	m^2_{xx}	m^2_{yy}	m^2_{xy}	m^2_{yx}
Value kg	–	–	–	–	–	–	–	–
Calculated	2.364	2.430	–0.069	–0.028	2.341	2.284	–0.027	–0.055
Error %	–	–	–	–	–	–	–	–

9 Summary and Conclusions

The article presents the sensitivity analysis of the method for the determination of 24 bearing dynamic coefficients. These coefficients are determined experimentally for a rotor system supported by two bearings, in a single operation by making use of the method of least squares. The 16 dynamic coefficients (including 4 stiffness, 4 damping, and 4 mass ones) are identified for each bearing at one time. For the purpose of verifying the sensitiveness of the key input parameters, a numerical model in Samcef Rotors program was created. The model containing a rotor supported by two bearings allowed to change the parameters which are difficult or even impossible to verify during experimental tests.

As part of this work, the calculations were carried out for a reference model of the rotor—symmetrical rotor with the disk located in the middle of the shaft, and the shaft was equipped with two bearings equidistant from its center. As the values of stiffness and damping coefficients as well as the shaft mass were known in advance, there was a possibility to make a direct comparison between the calculated values and actual values. After performing the calculations for the reference model, the maximum relative error concerning the stiffness coefficients amounted to 0.63 %. The same error in comparison to the damping coefficients did not exceed 3.76 %. In addition, it was noted that the main stiffness and damping coefficients were calculated with an accuracy two times higher relating to the cross-coupling stiffness and damping coefficients. The shaft mass can be identified with an accuracy of 0.003 %. Such good calculation accuracy allows to calculate the shaft mass, which in reality weighs 4.845 kg, with an accuracy of 0.0001 kg.

One of the parameters examined concerned shifting of the excitation force from the initial position located in the middle of the shaft by 30 mm from the disk. It turns out that even this small shift (5 % of the distance between the bearings) leads to the relative errors amounting to around 12, 15.5, and 0.2 % for the stiffness, damping and mass coefficients, respectively. A modification of the calculation algorithm was described, which involves an uneven distribution of the excitation force on each bearing. This modification improved the accuracy of the results obtained and the relative errors were at a level similar to errors calculated for the reference model. It was quite interesting to compare the calculated mass coefficients with their actual values (half of the shaft mass). In the case of initial version of the algorithm (without correction) the relative errors concerning these coefficients were around 15 %, while introducing the appropriate correction into the algorithm reduced these errors to approximately 3.8 %. By checking the values of mass coefficients, we are able to assess the correctness of the whole set of bearing dynamic coefficients in a preliminary stage. This example illustrates how convenient it is to carry out the calculation of all coefficients (not only stiffness and damping coefficients) in a single operation.

A well-defined excitation force in the calculation algorithm produces correct results of bearing dynamic coefficients. The force generated by an impact hammer is not always applied in the intended direction. In the course of the experimental research there is a possibility to make an "imprecise" impact, which leads to applying the force in an unintended direction. This was the reason why it was decided to analyze the case in which the angle of the excitation force F_x was changed by 15°. It is quite a large angle; however, it is worth checking what the discrepancy between the results this change entails. The calculations of bearing dynamic coefficients were carried out as if the excitation angle α was 0°. It turns out that such an "imprecise" impact, i.e., deviation of $\pm 15°$ from the intended direction of impact, results in the relative error amounting to 56 %. Such a high level of error appears only for certain coefficients. The highest levels of relative errors resulted from the identification of cross-coupling coefficients *YX*. The calculation of *YY* coefficients also produced significant errors. Other coefficients were determined with accuracy of 3 %.

Most often it is not possible to make direct measurements of the bearings in the course of experimental research. Therefore, the calculations of bearing dynamic coefficients were performed on the basis of the signal measured at a distance of 30 mm from the middle of the bearing. It turns out that the appropriate transformation of the signals ensures the possibility to get correct values of dynamic coefficients. Ignoring this shift entails an error of measurement amounting to around 2 %.

In the course of experimental research activities consisting of excitation the rotating shaft twice with an impact hammer, the impact of changes in shaft material stiffness on calculation results was also investigated. All previous calculations were carried out assuming that the shaft is an object made of steel. The analysis was then conducted in which the value of Young's modulus was 100 times higher than that of conventional steel. After this test, the calculation results of bearing dynamic coefficients have not changed. This means that the longitudinal modulus of elasticity of the shaft material does not affect the calculated stiffness and damping coefficients.

In order to verify the calculation results concerning asymmetrical rotor the base numerical model of the rotor has been modified. One free end of the shaft was shorter than the other one by 60 mm. This situation occurs when, for example, the installation of a coupling on one side of the shaft is necessary. The determination of the bearing dynamic coefficients obtained by simulation studies for such asymmetrical rotor can be considered satisfactory. The relative error values were very close to those from the reference model.

The examination of the results has led to the conclusion that some parameters do not affect the calculation results whereas the subsequent correction of the parameters which may have impact on the results is difficult. The article shows that certain parameters such as shifting of excitation force or different placement of measurement sensors result in measuring errors. However, it was demonstrated that their negative impact on the calculation results can be reduced by some minor corrections/improvements. The hints given in this article can be useful in experimental determination of bearing dynamic coefficients, and their detailed description gives some indication of the possibilities and limitations of the method itself.

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